Stat 1040, Fall 2014 **Name** ______________________
Test 2 **Recitation Leader** ____________ **Time** _____

1. An elementary school in Logan employs 15 teachers; 11 are women and 4 are men. Two teachers are selected at random to meet the governor and attend a reception in SLC.

a) Find the probability that both are women? (5 points)

$\frac{11}{15} \times \frac{10}{14}$

b) Find the probability that at least one is a woman? (5 points)

$1 - \frac{4}{15} \times \frac{3}{14}$

c) Find the probability that both are the same gender? (5 points)

$\frac{11}{15} \times \frac{10}{14} + \frac{4}{15} \times \frac{3}{14}$

2. A standard deck of cards is shuffled. You are dealt one card.

a) Find the probability that you get an *ace* or a *king*. (5 points)

$\frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \frac{2}{13}$

b) Find the probability that you get an *ace* or a *heart*. (5 points)

$\frac{4}{52} + \frac{13}{52} - \frac{1}{52}$

3. A pair of fair dice is rolled. If the sum is *seven* then you win \$25; otherwise, you lose \$5. You repeat this game 50 times.

a) What is the probability of rolling a *seven*? (5 points)

$\frac{6}{36}$, $\frac{1}{6}$

b) Construct an appropriate box model for determining your total winnings. (10 points)

$\boxed{25, -5, -5, -5, -5, -5}$ → Draw 50 & consider the sum of the draws.

4. Draw 400 times with replacement from the box $\left[\boxed{5}\ \boxed{7}\ \boxed{8}\ \boxed{9}\ \boxed{11}\right]$ and consider the sum of the draws.

AV = 8

a) What do you expect the sum of the draws to be? (5 points)

EV = Box AV × 400 = 3200

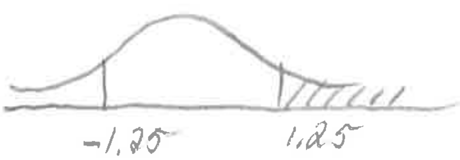
b) Find the SE for the sum of the draws. (Note: the box SD is 2) (5 points)

SE for sum = Box SD × $\sqrt{400}$ = 2 × 20 = 40

c) Find the probability that the sum of the draws is greater than 3250. (5 points)

Sum of draws follows normal curve :

$$\frac{3250-3200}{40} = \frac{50}{40} = 1.25$$



A(1.25) = 79% , 21% / 2

10.5% (or 11% is ok)

5. True or False: (5 points each)

T a) The correction factor used when sampling without replacement can be ignored if the population size is very large when compared to the sample size.

T b) If you draw 3600 times with replacement from $\left| -13\ ,\ 0\ ,\ \pi\ ,666 \right|$, the average of the draws will follow the normal curve.

T c) When drawing with replacement from a given box, as the number of draws increases, the SE for the sum of the draws increases but the SE for the average of the draws decreases.

T d) We want to estimate the percentage of adults in each state in the USA who have no health insurance. Using a simple random sample of size 1500 in California and constructing a 95% confidence interval will be just as accurate as using a sample of size 1500 in Utah to get a 95% confidence interval.

6. Health officials take a simple random sample of 2700 Utah elementary school children and find that 528 are deficient in vitamin D. Find a 95% confidence interval for the percentage of all Utah elementary school children who are deficient in vitamin D. (10 points)

$\boxed{?\ 1s,\ ?\ 0s}$ → Draw 2700 & consider the % 1s drawn. The % 1s drawn follows normal curve.

$$\frac{528}{2700} \times 100\% = 19.5\% \qquad SE \text{ for } \% = \frac{\text{Box SD} \times \sqrt{2700}}{2700} \times 100\%$$

$$\text{Box SD} \approx \sqrt{\frac{528}{2700} \times \frac{2172}{2700}} = .397$$

$$SE \text{ for } \% = \frac{(.397)\sqrt{2700}}{2700} \times 100\% = .764\%$$

$$19.5\% \pm 2(.764\%), \quad 19.5\% \pm 1.53\%$$

7. A financial advisement team conducted a simple random sample of 900 federal income tax forms filed for the year 2013 in which the individual received a tax refund. The average tax refund for the sample was $2,950. The SD for the sample was $1800.

a) Construct the 95% confidence interval for the average tax refund for all taxpayers receiving a refund. (10 points)

$\boxed{\text{tax refunds}}$ → Draw 900 & consider AV of draws. The AV of draws follows normal curve.

$$SE \text{ for } AV = \frac{\text{Box SD} \times \sqrt{900}}{900} \approx \frac{1800 \times 30}{900} = \$60$$

$$\$2950 \pm 2(\$60) \quad \$2950 \pm \$120$$

b) Construct the 90% confidence interval for the average tax refund for all taxpayers receiving a refund. (5 points)

$$\$2950 \pm (1.65)60, \quad \$2950 \pm \$99$$

PROBABILITY RULES (The World)

Definition: The probability [chance] of event A is the proportion [percentage] of the time A is expected to happen when the random process is repeated over and over again.

Opposite Event Rule: The probability that event A happens is equal to one minus the probability that A doesn't happen.

Multiplication Rule: The probability that events A and B both happen is equal to the probability that A happens times the probability that B happens given that event A has occurred.

Definition: Two events are mutually exclusive when the occurrence of one prevents the occurrence of the other.

Addition Rule: The probability that event A or event B happens is equal to the probability that A happens plus the probability that B happens minus the probability that both happen. If events A and B are mutually exclusive, then the probability that event A or B happens is simply the sum of the probabilities.

Definition: Two events are independent if when one happens, the probability that the other happens is unchanged.

Fundamental Counting Principle: If event A can occur in m ways and after A occurs event B can occur in n ways, then the number of ways both events A and B can occur is m x n .

The number of ways k objects can be selected from n objects without regard to order is

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Repeated Trials: Suppose we have n independent trials, and the probability that event E occurs in any given trial is p. Then the probability that E will occur exactly k times is $\frac{n!}{k!(n-k)!}p^k(1-p)^{n-k}$

Expected Value and Standard Error:

Suppose you randomly draw n times with replacement from a box.

EV for the sum of the draws = Box AV x n

SE for the sum of the draws = Box SD x $\sqrt{n}$

EV for the average of the draws = Box AV

SE for the average of the draws = $\frac{\text{Box SD} \times \sqrt{n}}{n}$

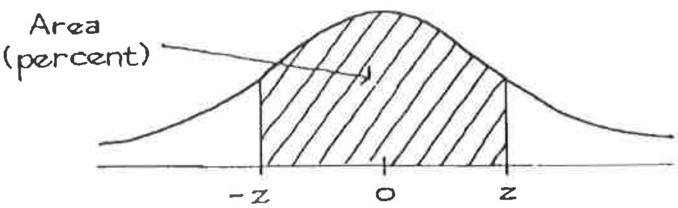
EV for the % of 1s drawn = % of 1s in the box

SE for the % of 1s drawn = $\frac{\text{Box SD} \times \sqrt{n}}{n} \times 100\%$

Short-Cut Formulas for SDs: *L-S* Boxes: Suppose the box contains only two different numbers, *L*s and *S*s with $L > S$, and the proportion of *L*s equal to p and the proportion of *S*s equal to $1-p$. Then

the Box SD $= (L-S)\sqrt{p \times (1-p)}$. For 0-1 boxes, the box SD $= \sqrt{p \times (1-p)}$ where p is the proportion of 1s and (1-p) is the proportion of 0s.

A NORMAL TABLE



z	*Area*	z	*Area*	z	*Area*
0.00	0	1.50	86.64	3.00	99.730
0.05	3.99	1.55	87.89	3.05	99.771
0.10	7.97	1.60	89.04	3.10	99.806
0.15	11.92	1.65	90.11	3.15	99.837
0.20	15.85	1.70	91.09	3.20	99.863
0.25	19.74	1.75	91.99	3.25	99.885
0.30	23.58	1.80	92.81	3.30	99.903
0.35	27.37	1.85	93.57	3.35	99.919
0.40	31.08	1.90	94.26	3.40	99.933
0.45	34.73	1.95	94.88	3.45	99.944
0.50	38.29	2.00	95.45	3.50	99.953
0.55	41.77	2.05	95.96	3.55	99.961
0.60	45.15	2.10	96.43	3.60	99.968
0.65	48.43	2.15	96.84	3.65	99.974
0.70	51.61	2.20	97.22	3.70	99.978
0.75	54.67	2.25	97.56	3.75	99.982
0.80	57.63	2.30	97.86	3.80	99.986
0.85	60.47	2.35	98.12	3.85	99.988
0.90	63.19	2.40	98.36	3.90	99.990
0.95	65.79	2.45	98.57	3.95	99.992
1.00	68.27	2.50	98.76	4.00	99.9937
1.05	70.63	2.55	98.92	4.05	99.9949
1.10	72.87	2.60	99.07	4.10	99.9959
1.15	74.99	2.65	99.20	4.15	99.9967
1.20	76.99	2.70	99.31	4.20	99.9973
1.25	78.87	2.75	99.40	4.25	99.9979
1.30	80.64	2.80	99.49	4.30	99.9983
1.35	82.30	2.85	99.56	4.35	99.9986
1.40	83.85	2.90	99.63	4.40	99.9989
1.45	85.29	2.95	99.68	4.45	99.9991